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Investigation of Direction of Arrival Estimation Using Characteristic Modes

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***Abstract*—A method is proposed to estimate the direction of arrival (DoA) of a traveling wave from characteristic mode weighting coefficients. These are obtained from the currents through the ports positioned on an antenna structure. The additional insight into the behavior of the antenna structure gained by the modal analysis is utilized to create a set of ports that allows to use a single conducting structure for the direction finding. It is shown that the proposed method works for a cubic antenna structure with 20 uncorrelated ports with good accuracy for any DoA.**

***Index Terms*—Antenna theory, characteristic modes, multi-mode antenna, angle of arrival, aircraft navigation.**

I. Introduction

In recent years, the popularity of the Theory of Characteristic Modes (TCM) has grown steadily over a wide range of application scenarios. However, the purpose of the greater part of the research conducted in this area is linked to the general demand for reliable, high data rate radio transmissions, like [1], [2] and [3]. In comparison, the field of radio direction finding in conjunction with the TCM has received less attention, although some results have already been presented. In particular, in [4] and [5] the authors utilize a multi-mode antenna designed using the TCM and employ the array interpolation technique and the wavefield modeling method to estimate the Direction of Arrival (DoA) for multiple incoming waves. An application example where the TCM is used in conjunction with a statistical model for DoA estimation has already been presented in [6].

The approach of this paper is directed towards the modal parameters and port specifications instead. These shall be utilized to show that antenna structures consisting of a single radiating element are suitable for DoA estimation by using the TCM. Due to their flexible geometry and compact size, these antenna structures are suited for many different applications, including unmanned aerial systems (UAS).

A brief introduction into characteristic modes is given in section II-A. For the proposed direction finding procedure, the current on an antenna structure caused by an incident plane wave has to be measured in samples provided by ports positioned on the structure. The port currents are then decomposed into modal port currents and an estimation of the modal weighting coefficients is obtained, as described in section II-B. The modal parameters will provide additional insight into the behavior of the structure and the ports. In section II-C it is shown how the DoA of the incident wave can be obtained from these modal weighting coefficients. Section III contains an example based on a cubic antenna structure.

II. Proposed Method

A. Characteristic Modes

The characteristic modes (CM) of a perfectly conducting body can be computed from the generalized eigenvalue equation [7]

$$\mathbf{X}\mathbf{I}_n = \lambda_n \mathbf{R}\mathbf{I}_n\,, \tag{1}$$

where $\mathbf{R}$ and $\mathbf{X}$ are the real and imaginary parts of the impedance matrix of the structure, respectively. The $N_{\max}$-by-$N_{\max}$ impedance matrix can be calculated from the Method of Moments (MoM) after applying a discrete mesh of $N_{\max}$ basis functions to the body. λ_n is the eigenvalue of the nth characteristic mode and $\mathbf{I}_n$ is the eigenvector of that mode.

If $\mathbf{I}$ is a vector containing current weights for an arbitrary current on the conducting body, normalized to radiate unit power, it can be expanded according to [8]

$$\mathbf{I} = \sum_{n=1}^{N_{\max}} b_n \mathbf{I}_n = \sum_{n=1}^{N_{\max}} \frac{\mathbf{I}_n^{\mathrm{H}}\mathbf{V}}{1+j\lambda_n}\mathbf{I}_n\,. \tag{2}$$

Here, b_n is the normalized modal weighting coefficient, $\mathbf{V}$ contains the weights to the normalized impressed electric field, j is the imaginary unit and $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose. We can rewrite (2) as a matrix-vector product

$$\mathbf{I} = \mathbf{I}_{\mathrm{CM}}\mathbf{b}\,, \tag{3}$$

where $\mathbf{b}$ is a column vector containing the normalized modal weighting coefficients b_n for all $N_{\max}$ characteristic modes and $\mathbf{I}_{\mathrm{CM}}$ is a matrix whose columns are the corresponding eigenvectors $\mathbf{I}_n$.

B. Decomposition of Port Currents

In order to be able to utilize the modal weighting coefficients for calculating the DoA in section II-C, these have to be determined from the total current on the antenna structure, which is excited by the incident plane wave. The total current weights $\mathbf{I}$ can be estimated by measuring the current through a finite number of lumped ports P on the structure. The current weight through port p is $I_{\mathrm{ports},p}$ and can be obtained from $\mathbf{I}$ by calculating the weighted sum of the current weights at the feed positions of the port. The column vector $\mathbf{I}_{\mathrm{ports}}$ contains the $I_{\mathrm{ports},p}$ for all ports and can be expressed in terms of the normalized modal weighting vector $\mathbf{b}$ by

$$\mathbf{I}_{\mathrm{ports}} = \mathbf{I}_{\mathrm{ports,CM}}\mathbf{b}\,. \tag{4}$$

It is evident from comparing (3) and (4) that the P-by-N_{max} matrix $\mathbf{I}_{\mathrm{ports,CM}}$ is obtained by extracting the modal port current weights $I_{\mathrm{ports,CM},p,n}$ from $\mathbf{I}_{\mathrm{CM}}$ for each port p and each characteristic mode n. To do this, the modal currents of the antenna structure have to be calculated beforehand using (1). The port-mode-matrix $\mathbf{I}_{\mathrm{ports,CM}}$ is independent of the excitation.

The system of linear equations (4) can now be solved for $\mathbf{b}$ to obtain the estimated normalized modal weighting vector $\mathbf{b}_{\mathrm{est}}$ from the port currents. Since the system of linear equations has to be solvable, the number of ports must be equal to or larger than the number of characteristic modes ($P \geq N$), which implies that in a real application with a limited number of ports, not all modes can be taken into consideration ($N < N_{\mathrm{max}}$). Moreover, the ports and therefore the rows of $\mathbf{I}_{\mathrm{ports,CM}}$ have to be linearly independent of each other, so each port provides new information. In order to lower the condition number of the matrix $\mathbf{I}_{\mathrm{ports,CM}}$ and thereby improving the numerical stability of the problem, the ports should also be orthogonal to each other. In general, the number and positioning of ports determines how well any current excited on the surface of the antenna can be measured. The TCM can be utilized to derive a suitable set of ports: If every port was only correlated with exactly one of the N most significant modes, any current that can be expressed as a superposition of these modes could in theory be calculated with perfect precision from the port currents. For currents that employ a higher number of characteristic modes, this setup provides an approximation. Returning to (4), it therefore becomes clear that reducing the number of modes used to create $\mathbf{I}_{\mathrm{ports,CM}}$ from N_{max} to N introduces an error that depends on the current excited by the specific incident plane wave. This leads in turn to a reduced accuracy of the estimated normalized modal weighting vector $\mathbf{b}_{\mathrm{est}}$. Consequently, selecting those ports and modes that allow for an optimal approximation of the modal weighting coefficients is important for the proposed method and will be further investigated in section III with the aid of an example.

C. Direction finding based on modal weights

The standard procedure for determining the direction of arrival of an incident plane wave is to employ an array of separate antenna elements and to calculate the DoA analytically from the array geometry, the antenna element patterns and the measured port currents [9]. Since the antenna ports in this work are all positioned on the same conducting structure, a different approach is taken here. In a first step, the total currents on the antenna structure are determined by simulation for a large number L of different incident plane waves, which arrive from uniformly distributed angles. Then, the port currents are extracted. It is assumed that the antenna and the ports are defined in such a way that in the investigated sector, the relationship between port currents and DoA is unambiguous. The estimated normalized modal weighting vector of the lth DoA $\mathbf{b}_{\mathrm{est},l}$ is calculated using (4). All acquired results are saved to an N-by-L reference matrix $\mathbf{B}_{\mathrm{s}}$.

Let us now assume a current on the investigated antenna structure was excited by a single electromagnetic plane wave incident from an unknown DoA. After the estimated normalized modal weighting vector $\mathbf{b}_{\mathrm{est}}$ is calculated, the estimated DoA can be obtained by calculating the correlation between $\mathbf{b}_{\mathrm{est}}$ and all columns of the previously determined reference matrix $\mathbf{B}_{\mathrm{s}}$ using

$$\rho(\mathbf{b}_{\mathrm{est}}, \mathbf{b}_{\mathrm{s},l}) = \frac{\mathbf{b}_{\mathrm{est}}^{\mathrm{H}} \cdot \mathbf{b}_{\mathrm{s},l}}{||\mathbf{b}_{\mathrm{est}}||\;||\mathbf{b}_{\mathrm{s},l}||}\,, \tag{5}$$

where $\mathbf{b}_{\mathrm{s},l}$ is the lth column of $\mathbf{B}_{\mathrm{s}}$. The DoA belonging to that $\mathbf{b}_{\mathrm{s},l}$ which has the highest correlation with $\mathbf{b}_{\mathrm{est}}$ is the resulting estimated DoA.

III. Demonstration using a Cube

The optimal direction finder for a general application is one that offers the same high precision for all possible plane wave directions of arrival and polarizations. It is clear that a spherical antenna structure is the theoretical optimum for a direction finder based on a single conducting body. However, in real world applications different geometries are preferred because they are easier to handle and manufacture. In this work, an ideal cube was chosen as an example structure to illustrate the proposed DoA estimation procedure. Due to the symmetry of the cube, it is possible to obtain data for both azimuth and elevation angles as well as for different polarizations.

As was stated in section II-B, using orthogonal ports is a promising approach for obtaining a good estimation of the modal weighting coefficients. In order to realize the ports, the design guidelines for symmetric multimode antennas presented in [10] are applied. These are based on the fact that each characteristic current can be assigned to exactly one irreducible representation of the symmetry group of the antenna geometry. The irreducible representations describe how the characteristic currents transform under the symmetry operations of the group. Due to the orthogonality theorem for irreducible representations [11], characteristic currents which belong to different irreducible representations are orthogonal to each other. If a port is designed in such a way that it fulfills the symmetry requirements of a given irreducible representation, it only excites those characteristic modes belonging to the same representation [10]. This way, orthogonal ports are realized, whose maximum number is governed by the number and the dimensions of the irreducible representations of the symmetry group. The symmetry group of the cube is the O_h group [11]. Based on this, a symmetry analysis reveals that the cube offers 20 orthogonal ports [10].

One possible implementation of these ports is shown in Fig. 1. One port consists of several symmetrically placed feeding points. The position of each lumped feeding point is indicated by the red line, while the blue arrow determines

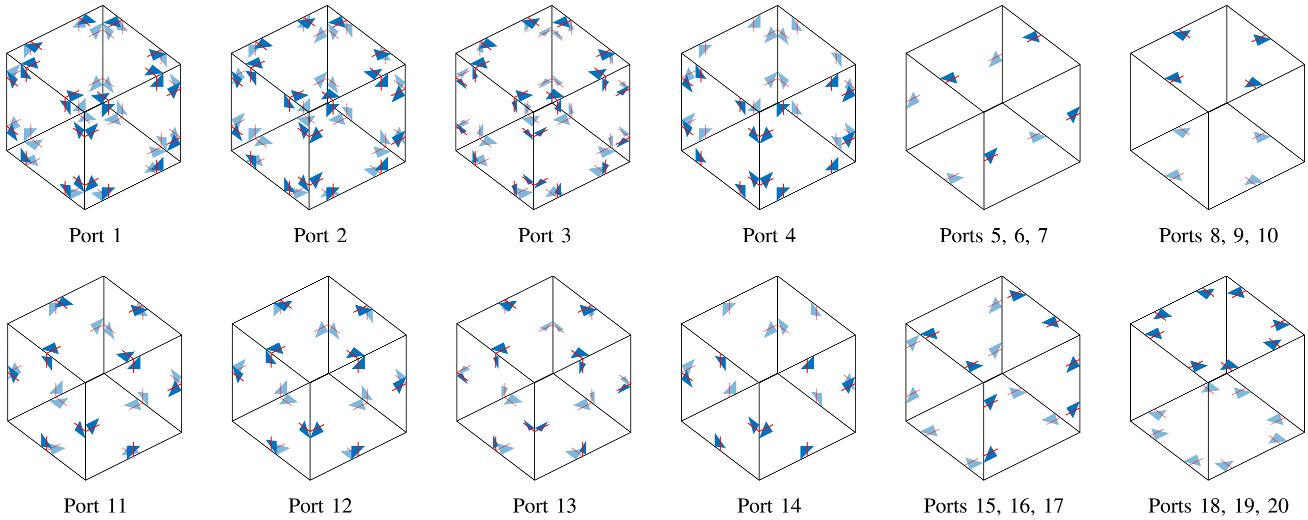

Fig. 1. Feed setups for 20 uncorrelated ports on a cube. Where multiple port numbers are given, the last two can be obtained from the displayed first one by rotating around the main axis. For ports 3 and 13, some feeding points only have half the amplitude of the other ports, which is marked with shorter arrows.

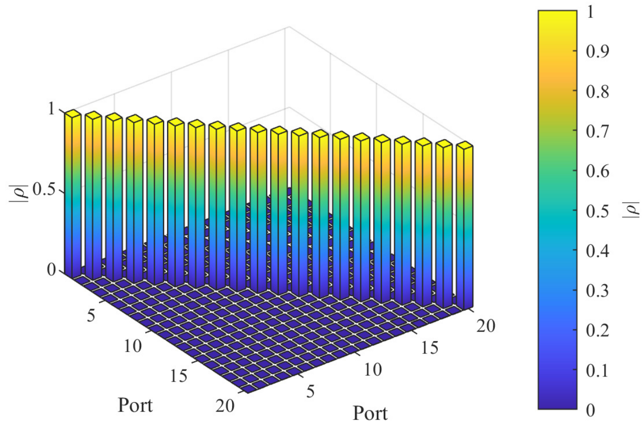

Fig. 2. Envelope correlation coefficient of ports.

the direction and amplitude of the feed. The feed amplitude is identical for all feeds of all ports, except for ports 3 and 13, where the feeds on the side surfaces are excited with half the amplitude of those on the top and bottom surfaces. For ports 5 to 14, only feeding points on the center of each cube edge are allowed while for the other ports, all feeds lie on the half way points between the start and the center of the edge. The envelope correlation coefficient of the far fields created by the lumped ports [10] is displayed in Fig. 2 for all lumped port combinations. As can be seen, there is no correlation between the far fields of any of the 20 realized ports. For this reason, we call these ports orthogonal.

We use a perfectly conducting cube with an edge length of a half wavelength, because the maximum number of orthogonal ports fits well with the number of characteristic modes that are close to being significant, while the structure remains relatively small. For aerial applications, signals at the frequencies $1030\,\mathrm{MHz}$ and $1090\,\mathrm{MHz}$ are already emitted by many aircraft due to the airborne collision avoidance system (ACAS) and can be utilized for direction finding. Therefore, their center frequency $1060\,\mathrm{MHz}$ is chosen as an example in this work, which results in an edge length of around $0.1414\,\mathrm{m}$.

It was shown in section II-B that the number of modes utilized to calculate the modal weights must not exceed the number of available ports. To get the best insight into the problem, the number of modes is chosen to be equal to the number of ports ($N = P = 20$). Since every characteristic mode current can be uniquely assigned to a single irreducible representation of the symmetry group [10], it can only be measured by a port that belongs to the same irreducible representation. This means that to utilize all ports, there has to be the same number of ports and modes for every irreducible representation. To illustrate this connection, we take a look at Fig. 4, where the modal significance (MS) in dB is displayed for the first 48 modes. When selecting the modes for the application, we start with the most significant mode 1 and assign it to its irreducible representation. For the next mode, we first check whether the number of modes already assigned to its irreducible representation is still below the dimension of the representation (meaning there is still an unoccupied port within that representation). This works perfectly for every mode up to mode 17. However, mode 18 belongs to the 9th irreducible representation Γ^9 of the O_h symmetry group, just like the modes 15, 16 and 17. Since Γ^9 has a dimension of three, there are three ports belonging to this representation (15, 16 and 17) and mode 18 would be the fourth mode that would have to be calculated from the information these three ports provide. Therefore, this mode cannot be used for the port-mode matrix $\mathbf{I}_{\mathrm{ports,CM}}$. The same principle leads to the

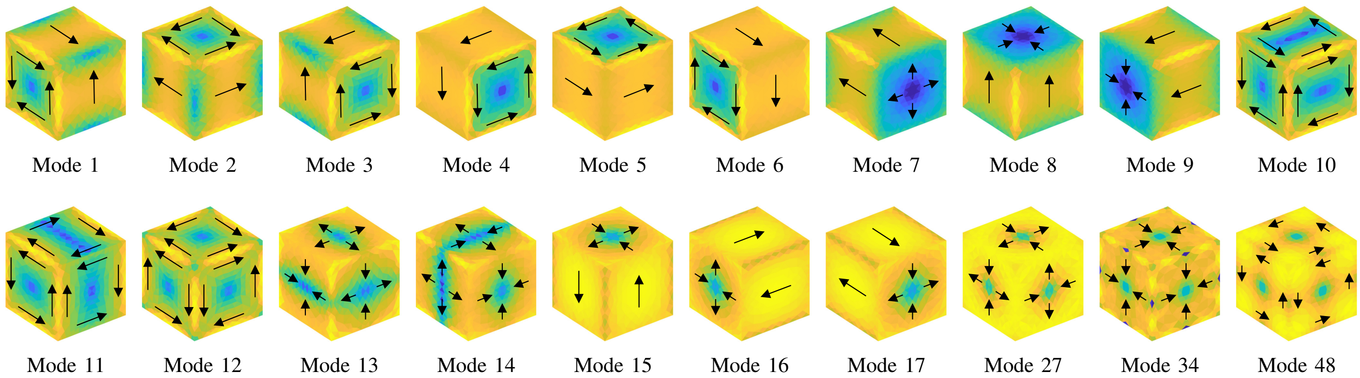

Fig. 3. Selection of characteristic mode current densities supported by the cube. For scale refer to the colorbar in Fig. 5

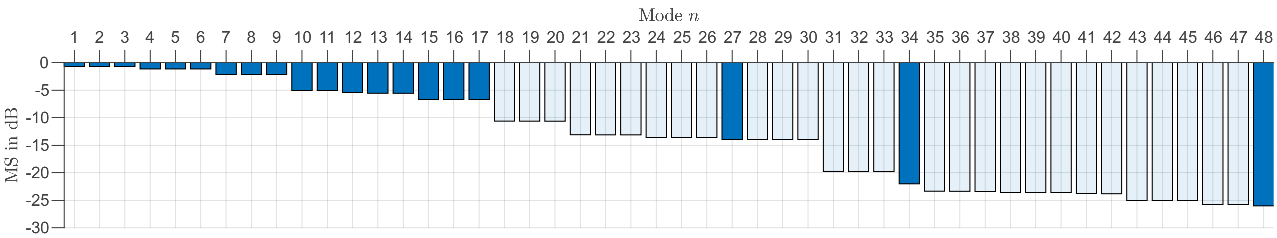

Fig. 4. Modal significance of the first 48 modes on the cube. Modes not used for the DoA estimation are displayed as transparent bars.

exclusion of all modes that are marked with transparent bars in Fig. 4. Finally, the modes 1 to 17, 27, 34 and 48 are selected. Their surface current densities are displayed in Fig. 3. Even though the higher order modes are not significant, simulations have shown that utilizing them increases the accuracy of the final DoA estimation in comparison to only using the first 17 modes.

After the modes are selected, the port-mode matrix can be calculated. The magnitude of its entries is shown in Fig. 5. It can be observed that most modes only interact with a single port. Modes 10 and 11 as well as 13 and 14 interact only with the two ports from their respective representation, which has no negative effect on the port orthogonality and thus the accuracy of the procedure. Modes 15 and beyond have some minor impact on the measurement results of ports from other representations, but this does not influence the condition number of the matrix significantly and therefore has no degrading impact on the numerical stability of the system of linear equations (4).

Since all parameters have been defined that are necessary to determine the estimated normalized modal weighing vector $\mathbf{b}_{\text{est}}$, the direction finding method described in section II-C can now be applied to the cubic antenna structure. Firstly, a spherical grid with a step width of 10 degrees in both azimuth and elevation is assembled around the cube. On every knot of the grid, a plane wave source directed towards the center of the cube is positioned. Here, only azimuthal polarization is considered to validate the general procedure. For each of these excitations, a method of moments simulation is run and the modal weights are estimated from the calculated lumped

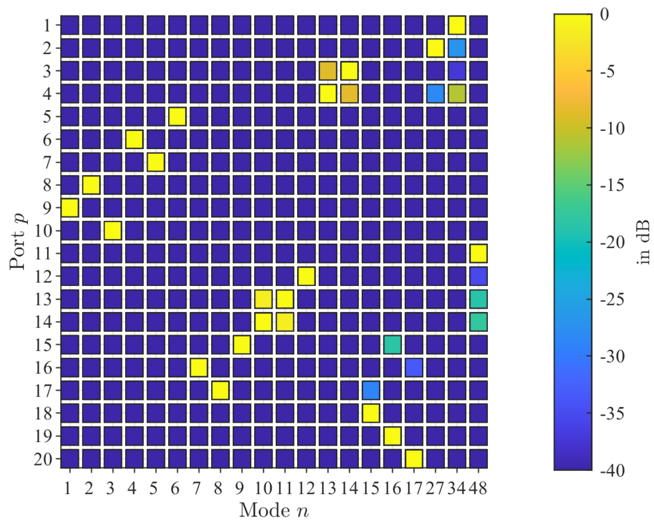

Fig. 5. Magnitude of the entries in the port-mode matrix $I_{\text{ports,CM},p,n}$ of the selected setup, normalized to the maximum value of each port.

port currents on the cube. After assembling the reference matrix $\mathbf{B}_{\text{s}}$, the performance can be tested.

In the following, an error is defined as the difference between the actual and the estimated direction of arrival. The errors for azimuth and elevation are combined by calculating the euclidean norm in degrees. When testing with any of the original DoAs used to assemble $\mathbf{B}_{\text{s}}$, no errors occur. This proves that the specific combination of selected geometry,

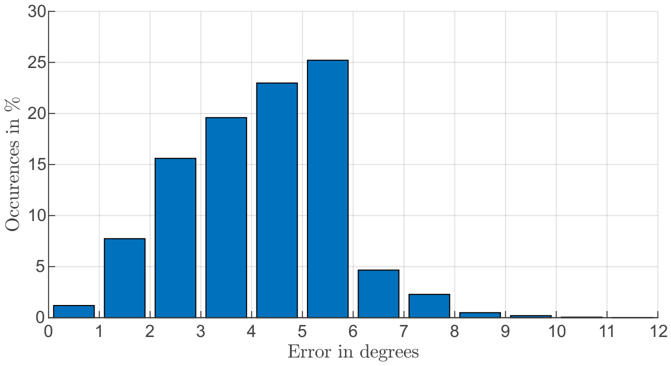

Fig. 6. Distribution of errors of the estimated DoA for the proposed example setup, obtained by using randomly distributed DoAs for testing. Based on the reference data, the accuracy is roughly 7 degrees.

ports and modes does not contain any ambiguity. For the second test, the testing DoAs are positioned in the center between the points of the spherical grid. Thereby, each one has a distance of $5\sqrt{2}$ degrees to each of the four neighboring grid knots. When the DoA estimation is executed, each testing DoA is matched to one of its four neighbors, which is the optimal behavior. A third and final test is carried out by using randomly distributed DoAs. On each $10^\circ \times 10^\circ$ patch of the grid, ten DoAs were randomly chosen from a uniform distribution. The distribution of the errors that occurred during this simulation is plotted in Fig. 6. Over 95% of the simulated DoAs resulted in errors lower than the theoretical maximum accuracy of $5\sqrt{2} \approx 7$ degrees, while some outliers lead to errors as high as 12 degrees. The relatively high maximum error shows that for a few DoAs, the limited number of utilized ports and modes limits the resolution of the procedure. However, most of the errors observed could be mitigated by reducing the step size of the spherical grid and thereby using more DoAs as reference to generate the matrix $\mathbf{B}_\mathrm{s}$. Overall, the proposed method provides good results for the example. This shows that it is possible to determine the DoA from modal parameters.

IV. Conclusion

A procedure was introduced that allows a direction of arrival estimation based on the weighting coefficients of the characteristic modes supported by a single conducting antenna structure. As an example, a cubic antenna structure with 20 ports and an edge length of a half wavelength was investigated. In simulations based on this example, the procedure worked with good accuracy. While the DoA estimation would also work without the modal weighting coefficients by using the port currents instead, the proposed approach offers additional insight into the behavior of the antenna. The knowledge gained from this approach was utilized to derive a port placement procedure for the multi mode antenna.

In future work, this knowledge shall be used to derive generalized design rules for multi mode antennas in direction finding applications.

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